35333/C 330

Reg. No. | | | | | | | | |

III Semester B.Sc.3 Degree Examination, Nov./Dec. 2016
MATHEMATICS (Optional)
Paper – I : Mathematical Logic and Real Analysis (Regular – w.e.f. 2015-16)
(Fresh and Repeater New Syllabus)

Time : 3 Hours　　　　　　　　　　　　　　　　　　　　Max. Marks : 80

Instructions : *1) Question paper contains* ***three*** *Parts* ***A, B, C.***
2) Answer ***all*** *questions.*

PART – A

1. Answer **any ten** of the following : **(10×2=20)**

a) Define conditional and biconditional of a proposition.

b) Give the direct proof of the statement : "The sum of two even integers is even".

c) Construct the truth table for $(p \rightarrow q) \vee (p \rightarrow \sim p)$.

d) If $u = x + y$ and $v = \frac{y}{x+y}$ then evaluate $\frac{\partial(u, v)}{\partial(x, y)}$.

e) Expand e^x cosy by Maclaurin's theorem as far as the second degree terms.

f) State the sufficient condition for extreme value for functions of two variables.

g) Find the stationary points of the function $f(x, y) = x^3 + y^3 - 3xy$.

h) Define Null sequence and give an example.

i) If $\{a_n\}_{n \in N}$ converges to 'a' then prove that $\{Ka_n\}_{n \in N}$ converges to 'Ka' where K is a constant.

j) Find the limit of the sequence $\{x_n\}_{n \in N}$ where $x_n = \frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \ldots + \frac{n}{n^2}$.

k) Prove that every Cauchy sequence is bounded.

l) Define limit point of a sequence and give an example.

P.T.O.

PART – B

Answer **any four** of the following : **(4×5=20)**

2. Define converse, inverse and contrapositive of a conditional proposition and also write its truth table.

3. With usual notations prove that $\frac{\partial(u_1, u_2)}{\partial(x_1, x_2)} = \frac{\partial(u_1, u_2)}{\partial(y_1, y_2)} \times \frac{\partial(y_1, y_2)}{\partial(x_1, x_2)}$.

4. Find the extreme values of $f(x, y) = xy(6 - x - y)$.

5. If $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$ then prove that $\lim_{n\to\infty} a_n . b_n = a.b$.

6. Discuss the convergence of the sequence $\{x_n\}_{n\in N}$ where
$x_n = 1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots + \frac{1}{3^n}$

7. State and prove Cauchy's second theorem on limits.

PART – C

Answer **any four** of the following : **(4×10=40)**

8. a) Define universal and existential quantifiers and explain with suitable examples.

   b) Prove that $(p \leftrightarrow q) \leftrightarrow [(\sim p \vee q) \wedge (\sim q \vee p)]$ is a tautology.

9. a) If $x + y + z = u$, $y + z = uv$ and $z = uvw$ then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.

   b) State and prove Lagrange's mean value theorem for functions of two variables.

10. a) Explain Lagrange's Method of undetermined multipliers to find the stationary values of the function $f(x, y, z)$ where x, y, z are connected by the relation $\phi(x, y, z) = 0$.

    b) Find the maximum value of the function $x^2y^2z^2$ subject to the condition $x^2 + y^2 + z^2 = a^2$.

11. a) Prove that every monotonic increasing sequence which is bounded above converges to its least upper bound.

b) Show that the sequence $\{x_n\}_{n\in N}$ defined by $x_1 = \sqrt{2}$ and $x_{n+1} = \sqrt{2x_n}$ converges to 2.

12. a) Prove that every convergent sequence is a Cauchy sequence.

b) Prove that :

i) $\lim\limits_{n\to\infty}\left(\dfrac{1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}}{n}\right)=0$

ii) $\lim\limits_{n\to\infty}\left[\dfrac{1}{\sqrt{n^2+1}}+\dfrac{1}{\sqrt{n^2+2}}+\dfrac{1}{\sqrt{n^2+3}}+\ldots+\dfrac{1}{\sqrt{n^2+n}}\right]=1.$
