12323/C 230

Third Semester B.C.A. Degree Examination, Nov./Dec. 2011
DISCRETE MATHEMATICAL STRUCTURES

Time : 3 Hours Max. Marks : 80

Instructions : *1) Attempt **any five full** questions.*
*2) **All** questions carry **equal** marks.*

I. 1) How many 3 digit numbers can be formed by using the digits 2, 3, 5, 6, 7, 9 repetitions not being allowed ?

i) How many of these are less than 400 ?

ii) How many of these are even ?

iii) How many of these are multiples of 5 ?

2) Find the number of permutations of the letters of the word ENGINEERING. In how many arrangements the 3E's are together ?

3) In how many ways can be distribute 12 identical pencils to 5 children so that every child gets at least 1 pencil ? **(8+4+4=16)**

II. 1) Find the coefficient of $x^2y^2z^3$ in the expansion of $(x + y + z)^7$.

2) Prove the logical equivalence using laws of logic

$$\neg[\neg[(p \vee q) \, n \, r] \vee \neg q] \Leftrightarrow q \wedge r$$

3) Test the validity of the following argument.

$$p \rightarrow r$$
$$\neg p \rightarrow q$$
$$q \rightarrow s$$
$$\overline{\therefore \neg r \rightarrow s}$$

(4+6+6=16)

P.T.O.

III. 1) Give an indirect proof of the statement "If x^2 is even, then x is even".

2) State any five rules of inference along with their names.

3) Give counter example to disprove the statement "only odd numbers are prime". **(4+10+2=16)**

IV. 1) Mention the two types of quantifiers. Explain these with examples.

2) For any two sets A & B prove that

i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$ ii) $A - B = A \cap \overline{B}$

3) State and prove multiplication theorem of probability. **(6+4+6=16)**

V. 1) Find the G.C.D. of 275 and 726 and express it as 275x + 726y.

2) Find the number and sum of all divisors of 960.

3) Prove by Mathematical Induction 1.2 + 3.4 + 5.6 + ______ + (2n – 1)

$2n = \frac{n(n+1)(4n-1)}{3}$.

4) Write the explicit formula for the sequence – 4, 16, – 64, 256, ...

(4+4+6+2=16)

VI. 1) In a class of 52 students 30 are studying Mathematics cause, 28 are studying business course and 13 are studying both the courses. How many in this class are studying at least one of these courses ? How many are studying neither of these courses ?

2) If one integer is selected at random from among the integers 1 to 15. If A is the event that a number selected is even and B is the event that a number selected is divisible by 3. Find $P(A \cup B)$.

3) Prove that

i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ **(6+4+6=16)**

VII.1) Let A = {a, b, c, d } and let R be the relation on A that has the matrix.

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Construct the diagraph of R and list in-degrees and out-degrees of all vertices.

2) If R is a relation as A = $\{a_1, a_2, \ldots\ a_n\}$, then prove that $M_{R^2} = M_R\, M_R$.

3) Show that congruence mod2 is an equivalence relation. **(6+5+5=16)**

VIII. 1) If f : A $\rightarrow$ B is a bijection, then prove that $f^{-1} : B \rightarrow A$ is also bijection.

2) Let f and g be two functions from R to R defined by $f(x) = x^3$ and $g(x) = x^2 + 1$. Find gof and fog.

3) If f : A $\rightarrow$ B and g : B $\rightarrow$ C are invertible functions then prove that $(gof)^{-1} = f^{-1}\, og^{-1}$.

4) Define Pigeonhole principle. Show that if any five numbers from 1 to 8 are chosen, then two of them add upto 9. **(4+4+4+4=16)**