Reg. No. [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ]

III Semester B.C.A. 2 Degree Examination, November/December 2017
DISCRETE MATHEMATICAL STRUCTURES

Time : 3 Hours Max. Marks : 80

***Instructions** : 1) Scientific calculators are **allowed**.*
*2) Answer **all** questions.*

PART – A

I. Answer **any ten** questions. **(2×10=20)**

1) If U = {a, b, c, d, e, f, g, h} and A = {a, b, c} and B = {c, d, e} then find $\bar{A} \cap \bar{B}$.

2) Define combination with example.

3) Find the number of permutations of the letters of the word "FATE".

4) Construct the truth table for $(p \wedge q) \leftrightarrow (p \vee q)$.

5) Define existential quantifier.

6) State well ordering principle.

7) Find the number of positive divisors of 756.

8) Define reflexive relation.

9) If A = {1, 2, 3, 4} and B = {a, b, c} find $B \times A$.

10) Let A = {2, 3, 4, 5} and R be relation on A defined by aRb if and only if $a < b$.

11) Let a function $f : R \rightarrow R$ defined by $f(x) = x^2 + 2x + 2$. Determine the image of the subset $A_1 = \{1, 3\}$ of R .

12) Consider the functions f and g defined by $f(x) = x^2$ and $g(x) = x^3 + 1$ $\forall x \in R$ find $g_\circ f$.

PART – B

II. Answer **any six** questions. **(6×5=30)**

13) A party is attended by n persons. If each person in the party shakes hands with all the others in the party, find the number of hand-shakes.

14) For any three sets A, B and C prove that $A \cup (B \cup C) = (A \cup B) \cup C$.

15) Prove that the following proposition is a tautology $[(p \to q) \wedge (q \to r)] \to (p \to r)$.

16) Give direct proof of the statement "If n is an odd integer then, n + 11 is an even integer".

17) Prove by the method of mathematical induction that

$$1^2 + 2^2 + 3^2 + ...+ n^2 = \frac{n(n+1)(2n+1)}{6} \quad \forall n \in N.$$

18) Find the GCD of 495 and 675 and express it in the form 495m + 695n.

19) If A = {1, 2, 3, 4} and R = {(1, 2), (1, 3), (2, 4), (3, 2), (3, 3), (3, 4)} be a relation on A find R^2 and draw the diagraph of R^2.

20) If R is a relation on $A = (a_1, a_2, ..., a_n\}$ then prove that $M_{R^2} = M_R \odot M_R$.

PART – C

III. Answer **any three full** questions.

21) a) A committee of 8 members is to be choosen from 9 teachers and 4 students. In how many ways can this be done if there are atmost 6 teachers ?

b) A fair die is thrown twice. Find the probability that

i) An even number occurs in atleast one throw .

ii) Even numbers occur on both throws. **(5+5=10)**

22) a) State any five rules of inference along with their names.

b) Prove that $(p \vee q) \wedge \neg(\neg p \wedge q) \Leftrightarrow p$ using laws of logic. **(5+5=10)**

23) State and prove the fundamental theorem of arithmetic. **10**

24) a) Let A = {1, 2, 3, 4} and B = {a, b, c}

let R = {(1, a), (1, b), (2, b), (2, c), (3, b), (4, a)}

and S = {(1, b), (2, c), (3, b) (4, b)}

Compute :

i) $\bar{R} \cup \bar{S}$

ii) S^{-1}

b) Show that congruence of mod2 is an equivalence relation. **(5+5=10)**

25) a) If f : A $\rightarrow$ B is a bijection prove that f^{-1} : B $\rightarrow$ A is also bijection.

b) Let $f(n) = 3n^4 - 5n^2$ and $g(n) = n^4$ be defined for positive integers n. Then show that f and g have the same order. **(5+5=10)**